## KENDRIYA VIDYALAYA SANGATHAN GUWAHATI REGION HALF YEARLY EXAMINATION 2018-19

CLASS: XII SUBJECT: MATHEMATICS

## **General Instructions: -**

1. All Questions are compulsory.

2. This Question Paper contains 29 questions.

3. Section A contains 4 Questions carrying 1 mark each.

4. Section B contains 8 Questions carrying 2 marks each.

5. Section C contains 11 Questions carrying 4 marks each.

6. Section D contains 6 Questions carrying 6 marks each.

Section A		
		Marks
Q1.	Find the value of a and b for which $\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ .	1
Q2.	State where the function $f: N \rightarrow N$ given by $f(x) = 5x$ is injective.	1
Q3.	Let * be the binary operation on N given by a * b = HCF (a,b) a, $b \in N$ . write the value of 22 * 4.	1
Q4.	If $ \vec{a} + \vec{b}  =  \vec{a} - \vec{b} $ , show that $\vec{a}$ and $\vec{b}$ are perpendicular.	1
Section B		
Q5.	The side of an equilateral triangle is increasing at the rate of 0.5 cm/sec. Find the rate of increase of the perimeter.	2
Q6.	Find the angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ if $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$ and $\vec{b} = 3\vec{i} + \vec{j} - 2\vec{k}$ .	2
Q7.	If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ , prove that $A^3 - 4 A^2 + A = 0$ .	2
Q8.	Solve: $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$ .	2
Q9.	Evaluate: $\int_{1}^{2} \left(\frac{x-1}{x^{2}}\right) e^{x} dx.$	2
Q10.	If y = sin <sup>-1</sup> x show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$	2
Q11.	Solve the differential equation $\frac{dy}{dx} = y(e^x+1)$ .	2
Q12.	Evaluate $\int_{0}^{\frac{\pi}{2}} \frac{\sin^{4}x}{\sin^{4}x + \cos^{4}x} dx$	2
Section C		
Q13.	Find the constant a and b so that the function 'f' defined below is continuous at 3 and 5 $f(x) = \begin{cases} 1, x \le 3 \\ ax + b, 3 < x < 5. \\ 7, \ge 5 \end{cases}$	4
Q14.	Differentiate the following w.r.to x: $y = x^{\cos x} + \cos x^{x}$ .	4
Q15.	For the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ , find the interval in which $f(x)$ is increasing and decreasing.	4
Q16.	Evaluate: $\int \frac{1+\cot x}{x+\log \sin x} dx.$	4
Q17.	Consider the binary operation $*$ on the set {1, 2, 3, 4, 5} defined by minimum (a, b) . Write the operation table of the operation $*$ .	4
Q18.	If $\vec{a} = \vec{i} - \vec{j} + 2\vec{k}$ and $\vec{b} = 2\vec{i} + \vec{j} - \vec{k}$ , find $(2\vec{a} - \vec{b}) \times (\vec{a} + 2\vec{b})$ .	4
Q19.	Show that the function f: $R \rightarrow R$ defined as $f(x) = x^2$ is neither one –one nor onto.	4

TIME: 3 HRS MM: 100

Q20.	Sum that the height of the cylinder of maximum volume that can be inscribed in a	4
	sphere of radius R is $\frac{2R}{\sqrt{3}}$ .	
Q21.	Solve the differential equation $(x^3+y^3)dy - x^2y dx = 0$ .	4
Q22.	$ cosec^2x  cot^2x  1 $	4
	Without expanding , show that $\begin{vmatrix} cot^2x & cosec^2x & -1 \end{vmatrix} = 0$	
Q23.	Find the area of the region { (x, y) : $x^2+y^2 \le 4$ , $x + y \ge 2$ }.	4
Section D		
Q24.	Determine which of the following binary operation on the set N are associative and which are	6
	commutative.	
	i. $\vec{a} * \vec{b} = 1, \forall a, b \in \mathbb{N}$	
	ii. $\vec{a} * \vec{b} = \frac{a+b}{2}, \forall a, b \in \mathbb{N}$	
Q25.	Solve using matrices $x - y + z = 1$	6
	2x+y -z =2	
	x - 2y - z = 4.	
Q26.	Using properties of determinants, prove that the following	6
	$\begin{vmatrix} 3a & -a+b & -a+c \end{vmatrix}$	
	$\begin{vmatrix} a-b \\ 3b \end{vmatrix} = 3(a+b+c)(ab+bc+ac).$	
027	a-c b-c  $3c $	6
Q27.	If the lengths of the transmission when it is maximum	0
0.20	ind the area of the trapezium when it is maximum.	6
Q28.	Draw a rough sketch and find the area of the region bounded by the two parabolas y <sup>2</sup>	6
	= 4x and $x^2$ = 4y by method of integration.	
Q29.	Evaluate: $\int \frac{2x+1}{\sqrt{x^2+4x+3}} dx.$	6
Or		
	Evaluate the following definite integrals as limit of sums: $\int_{1}^{4} (x^2 - x) dx$ .	

[END]